THE UNIVERSITY OF SYDNEY

MATH2902 Vector Spaces

(http://www.maths.usyd.edu.au/u/UG/IM/MATH2902/)

Semester1, 2001 Lecturer: R. Howlett

Tutorial 9

1. Compute the given products of permutations.

$(i)\quad \begin{bmatrix}1&2&3&4\\2&1&4&3\end{bmatrix}\begin{bmatrix}1&2&3&4\\2&3&4&1\end{bmatrix}\qquad (ii)\quad \begin{bmatrix}1&2&3&4&5\\4&1&5&2&3\end{bmatrix}\begin{bmatrix}1&2&3&4&5\\5&4&3&2&1\end{bmatrix}$

$(iii)\quad \left(\begin{bmatrix}1&2&3&4\\2&1&4&3\end{bmatrix}\begin{bmatrix}1&2&3&4\\2&3&4&1\end{bmatrix}\right)\begin{bmatrix}1&2&3&4\\4&3&1&2\end{bmatrix}$

$(iv)\quad \begin{bmatrix}1&2&3&4\\2&1&4&3\end{bmatrix}\left(\begin{bmatrix}1&2&3&4\\2&3&4&1\end{bmatrix}\begin{bmatrix}1&2&3&4\\4&3&1&2\end{bmatrix}\right)$

2. Calculate the parity of each permutation appearing in Exercise 1.

3. Use row and column operations to calculate the determinant of

$$\begin{pmatrix}1&5&11&2\\2&11&-6&8\\-3&0&-452&6\\-3&-16&-4&13\end{pmatrix}$$

4. For each permutation $\sigma\in S_n$ define P_σ to be the $n\times n$ matrix with (i,j)-entry equal to 1 if $i=\sigma(j)$ and 0 if $i\neq\sigma(j)$. Prove that $P_\sigma P_\tau = P_{\sigma\tau}$ for all σ, $\tau\in S_n$.

5. What is the determinant of the matrix P_σ defined in Exercise 4?

6. Consider the determinant

$$\det\begin{pmatrix}1&x_1&x_1^2&\dots&x_1^{n-1}\\1&x_2&x_2^2&\dots&x_2^{n-1}\\\vdots&\vdots&\vdots&&\vdots\\1&x_n&x_n^2&\dots&x_n^{n-1}\end{pmatrix}.$$

Use row and column operations to evaluate this in the case $n=3$. Then do the case $n=4$. Then do the general case. (The answer is $\prod_{i>j}(x_i-x_j)$.)

7. Let $p(x)=a_0+a_1x+a_2x^2$, $q(x)=b_0+b_1x+b_2x^2$, $r(x)=c_0+c_1x+c_2x^2$. Prove that

$$\det\begin{pmatrix}p(x_1)&q(x_1)&r(x_1)\\p(x_2)&q(x_2)&r(x_2)\\p(x_3)&q(x_3)&r(x_3)\end{pmatrix}=(x_2-x_1)(x_3-x_1)(x_3-x_2)\det\begin{pmatrix}a_0&b_0&c_0\\a_1&b_1&c_1\\a_2&b_2&c_2\end{pmatrix}.$$