General Sessions

Australian Mathematical Society Annual Meeting
University of Sydney, September 1998.

Titles and abstracts of the talks

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Integrable actions and transformation group C^*-algebras with bounded trace

Astrid an Huef (Dartmouth College, NH, USA)

Let (G, X) be a second countable locally compact transformation group. If the action of G on X is free we present sufficient and necessary conditions for $C_0(X) \times G$ to have bounded trace. These conditions are closely related to the notion of integrable actions on $C_0(X)$ proposed by Rieffel.

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Functions of bounded variation and operator theory

Brenden Ashton(University of New South Wales)

For many types of operators there is a strong link between structure theorems for the operators and their functional calculus properties. For example the well-bounded operators have a functional calculus over absolutely continous functions of one variable. Another example is the AC operators which have a functional calculus over absolutely continous functions of two variables with a rectangle domain. In a similar setting to the two above examples we give a definition for absolutely continous functions of two variables with arbitrary compact domain. We then look at some of the properties of the operators which have a functional calculus with this new definition.

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Some aspects of homotopy theory of modules

Bea Bleile (University of New England)

Given a unital ring Λ, the notions of projective and injective Λ-modules are dual to each other. One obtains two different notions of homotopy in the category of Λ-modules depending on which of these notions is chosen as a starting point. The first part of this talk explains the above statements. In the second part the homotopy groups of Λ-modules are defined and double resolutions of Λ-modules are used to produce a long exact sequence connecting the homotopy groups and Ext groups. Finally, in the third part, the homotopy groups of finite cyclic groups are calculated both for injective and projective homotopy theory.

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A push-out construction for groups with operators

Imre Bokor (speaker, University of New England) and Peter Hilton

A satisfactory theory of localisation of crossed modules and of relative localisation both require the imposition of additional conditions which are not obviously natural. A setting is presented which renders the crucial constructions in both of the above situations functorial without requiring additional conditions.

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Extending the use of lie point symmetries for PDE

P. Broadbridge (University of Wollongong)

Since there are several freely available symmetry-finding algorithms based on computer algebra packages, there are advantages in incorporating ad hoc PDE solution methods in the classical Lie symmetry algorithm. Many solutions for topical PDEs, previously thought to be unrelated to symmetry, are in fact invariant under some classical Lie point symmetry. Define a standard symmetric differential equation to be any DE that has a Lie point symmetry that modifies at least one independent variable. Broadbridge and Arrigo prove that every solution of every linear standard symmetric PDE of order 2 or higher, is in fact invariant under some classical Lie point symmetry. These PDEs include all constant-coefficient equations and many other linear equations of topical interest. We illustrate this with a familiar Fokker-Planck equation. Knowing only one symmetry-generating vector field, we can construct not just a 2-dimensional subspace of similarity solutions, but an infinite dimensional space of higher- generation similarity solutions. A simple proof shows that this can be achieved using any symmetry of any linear standard symmetric PDE.

If we immerse a target nonlinear PDE in a larger system including differential constraints, then the symmetry algebra may be enlarged, allowing additional symmetry reductions and similarity solutions. Goard and Broadbridge have used symmetry-enhancing constraints to obtain solutions to the cylindrical boundary layer equations that were thought to be unrelated to symmetry. In another example, a 3+1 dimensional Navier-Stokes momentum equation with order-h correction is found to be equivalent to the linear Schroedinger equation when a constraint is added.

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The decomposition of groups of units of rings

Clare Coleman (University of Sydney)

In a finite commutative ring R the group of units decomposes as $G(R) = H \times J$ where $J = \{\, 1 + x \,|\, x \in \mathcal{J}(R) \,\}$ and $\mathcal{J}(R)$ is the Jacobson radical of R. We will give some examples of this decomposition occurring, and the corresponding results in rings without identity. In particular we will look at the decomposition for a special class of Munn rings, the definition of which will be given in the talk. We also give an example to show that not all rings decompose in this way.

Solving the word problem and the conjugacy problem in the singular braid monoid

Ruth Corran (Sydney)

In Vassiliev's theory of knot invariants, singular knots play a significant role. It has been shown (Birman, Armand-Ugon et al.) that every singular knot (in fact, singular link) can be represented by a closed singular braid, and a presentation for the monoid of singular braids has been given (Baez, Birman). We show that the (positive) singular braid monoid is a member of a class of monoids determined by their presentations, with the property that whenever common multiples exist, a unique least common multiple exists. A normal form for members of this class is given, thus solving the word problem. A solution to the conjugacy problem in the singular braid monoid is also given.

A surprise in the theory of well-bounded operators

Ian Doust (University of New South Wales)

The spectral theory of well-bounded operators on nonreflexive Banach spaces has always been a little complicated. On such spaces, a well-bounded operator $T \in B(X)$ admits an integral representation of the form

$$\langle Tx, x^* \rangle = b\langle x, x^* \rangle - \int_a^b \langle x, E(\lambda)x^* \rangle \, d\lambda, \qquad x \in X,\ x^* \in X^*.$$

Here $\{E(\lambda)\}$ is a family of projections acting on the dual space X^*. This isn't particularly satisfactory, so in the 1960s several subclasses of well-bounded operators were introduced for which one could find a suitable family of projections on X. Examples showed that these classes were all distinct. Recently (with Cheng Qingping) we have found that the examples are in error and indeed have shown that the picture is rather simpler than was previously imagined.

Convolution of univalent and multivalent functions and its applications in geometric function theory

R. M. Goel (Punjabi University, India)

The convolution of Hadamard product of two power series

$$f(z) = \sum_{n=1}^{\infty} anz^n \ , \quad g(z) = \sum_{n=1}^{\infty} b_n z^n$$

convergent in the unit disc $E = \{z : |z| < 1\}$ is defined by

$$(f * g)(z) = \sum_{n=1}^{\infty} a_n b_n z^n$$

which is convergent in E. The integral convolution of $f(z)$ and $g(z)$ above is defined by

$$(f \odot g)(z) = \sum_{n=1}^{\infty} \frac{a_n b_n}{n} z^n \ .$$

In this paper we discuss some geometric properties of certain sub-classes of analytic functions which remain invariant under Convolution. We show that some properties of certain sub-classes of univalent and multivalent functions are preserved under certain linear operations. We also give criteria for univalence and p-valence of anlytic functions. We discuss neighbourhoods of certain sub-classes of univalent functions. We define the principle of duality and give its applications in solving several extremal problems in geometric function theory.

Towards a generalized Euler's constant

Leon Hardy (Armidale)

An existence theorem for a generalized Euler's constant is given for positive, real orders of integration. The classical definition of integer order Euler's constant is extended, yielding a formula to compute Euler's constant for a given integration order. Euler's constant for integration order 1/2, whose value is 0.703234, is given as an example. Furthermore, a geometrical interpretation of the generalized Euler's constant and the fractional integration process is discussed.

A method of computing polynomial invariants of semisimple groups

Alexandre Iltiakov (Sydney)

We shall discuss a method of computing explicitly polynomial invariants of a system of several vectors in a complex vector space V with respect to a semisimple algebraic subgroup of $GL(V)$.

Finiteness and bounds for the number of overrings of Prüfer domains

A. Jaballah (Department of Mathematics, The University of Waikato)

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Let R be an integral domain, and let K be its field of fractions. A ring S that contains R and is contained in K is called an overring of R. The following two questions are still open. 1. When is the set of overrings of R finite? 2. If this set is finite, How many elements does it have.

The main purpose of this talk is to give some reasonable answers to the above questions in the case where R is a Prüfer domain.

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Maximum topological entropy of permutations and cycles

Deborah King (La Trobe University)

A finite fully invariant set of a continuous map of the compact unit interval induces a permutation of the invariant set. It is known that Misiurewicz- Nitecki cycles attain maximum entropy amongst all permutations of odd period. We define a family of permutations of length $2n (n \in N, n \geq 2)$ and a family of cycles of length $4n$, and give a brief summary of how to show that these families attain maximum entropy amongst all permutations (respectively cycles) of the same order.

Generalised vector products and their applications

Muhammad Arif and Abdul Majeed (Lahore)

The generalised vector product of two or more vectors in $\mathbb{R}^n$, $n \geq 4$, is defined.

Some applications of these concepts to differential geometry, Lie algebras, universal algebras and mechanics will be discussed.

A generalisation of the lower radical class

Robert McDougall (Central Queensland University, Rockhampton)

In this work we demonstrate that the lower radical class construction on a homomorphically closed class of associative rings generates a radical class for any class of associative rings. A new description of the upper radical class using the construction on an

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Locally ordered bisets and the Easdown-Hall representation

Brett McElwee

Nambooripad introduced the biordered set of a semigroup, or simply biset, as a certain partial algebra comprising the idempotents of that semigroup. The Easdown-Hall representation is a representation of a biset in terms of partial transformations of its $\mathcal{L}$ and $\mathcal{R}$-classes. This naturally generates a semigroup which is termed the Easdown-Hall semigroup of the underlying biset.

The biset of the Easdown-Hall semigroup of a biset E is typically larger than E, and so the representation lends itself to the construction of concrete bisets from smaller bisets. It is demonstrated that if E is locally a poset, a so-called locally ordered biset, then the biset of its Easdown-Hall semigroup is locally a semilattice.

This leads to an alternate proof of Nambooripad's characterisation of the bisets of regular locally inverse semigroups, and a construction of all normal bands.

Finally, posets are examples of bisets, and the Easdown-Hall semigroup is shown to be minimal, in a certain arrow-theoretic sense, when restricted to posets.

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Constant line-sum matrices and the wrong permutation

Gerry Myerson (Centre for Number Theory Research, Macquarie University)

Let A be a square matrix with nonnegative integer entries and all row sums and all column sums equal. By Birkhoff's Theorem, A can be expressed as a linear combination of permutation matrices with positive integer coefficients. Write $\beta(A)$ for the minimum, over all such expressions for A, of the number of distinct permutation matrices used. It is known that if A is $n \times n$ then $\beta(A) \leq n^2 - 2n + 2$, and this result is sharp.

Let P be a permutation matrix which appears as a summand in some expression for A. Trivially, $\beta(A - P) \geq \beta(A) - 1$. We prove that $\beta(A - P) \leq \beta(A) + n - 1$, and we show that for every n there exists A and P for which equality holds.

Semigroup actions on directed graphs

David Pask (University of Newcastle)

If a semigroup acts on a directed graph E, then it induces an action on the associated graph C^*-algebra, $C^*(E)$. In this talk I shall outline joint work with Trent Yeend, in which we prove an extension of the fundamental result of Gross and Tucker concerning free group actions on directed graphs to the semigroup case. If there is time I shall discuss how this result may be used in studying the semigroup C^*-dynamical systems which arise.

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Some aspects of Carmichael's conjecture

Walid Amin Ramadan-Jradi (University of Technology, Sydney)

Let ϕ denote Euler's function; that is, $\phi(x)$ is the number of natural numbers not exceeding x and relatively prime to x. It is more than ninety years since Carmichael conjectured that, for any natural number A, the equation $\phi(x) = A$ never has a unique solution [1]. Since then this conjecture has received a considerable amount of attention but few theoretical results have been obtained (see Hagis [2] and Pomerance [3]). Recent computer searches showed that Carmichael's Conjecture is valid below $10^{10,000,000}$ (Schlafly and Wagon [4]).

While mathematicians are still looking for more prime divisors of a counterexample to this conjecture, a new approach is offered in this talk to deal with this problem and its theoretical aspects. The purpose of this talk is to look at the conditions on Carmichael's Conjecture with an approach which relies on the concept of the set of solutions of $\phi(x) = A$.

References

[1] R.D. Carmichael, "Note on Euler's phi-function," *Bull. Amer. Math. Soc.* 28 (1922), 109–110.

[2] P. Hagis, "On Carmichael's Conjecture concerning the Euler's phi-function," *Boll. Un. Mat. Ital.* (6) 5–A (1986), 409–412.

[3] Carl Pomerance, "On Carmichael's Conjecture," *Proc. Amer. Math. Soc* 43 (1974), 297–298.

[4] S. Schlafly and S. Wagon, "Carmichael's Conjecture is valid below $10^{10,000,000}$," *Math. Comp.* 63 (1994), 415–419.

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Studying Heegaard stucture using Dehn filling techniques

Yo'av Rieck (University of Melbourne)

(Joint work with Eric Sedgwick, Oklahoma State University)

We study handlebody decompositions (or Heegaard structures) of 3-manifolds obtained by attaching a solid torus to T, a torus boundary component of a manifold X (Dehn Filling). We show that for all but finitely many manifolds thus obtained the handlebody decomposition of minimal complexity is the same as that of X, or obtained from it by a single standard move (destabilisation).

We than go on to show that a similar result holds without the assumption of minimal complexity, provided we assume that T is the only closed essential surface in X. This assumption is essential. We show:

All but finitely many manifolds obtained by filling X contain only finitely many Heegaard surfaces that are not surfaces for X, and these surfaces are obtained from a Heegaard surface for X by a single stabilisation.

Similarity, attraction and initial conditions in an example of nonlinear diffusion

A. J. Roberts and S. A. Suslov (University of Southern Queensland)

Similarity solutions play an important role in many fields of science. The recent book of Barenblatt (1996) discusses many examples. Often, outstanding unresolved issues are whether a similarity solution is dynamically attractive, and if it is, to what particular solution does the system evolve. By recasting the dynamic problem in a form to which centre manifold theory may be applied, based upon a transformation by Wayne (1994), we may resolve these issues in many cases. For definiteness we illustrate the principles by discussing the application of centre manifold theory to a particular nonlinear diffusion problem arising in filtration. Theory constructs the similarity solution, confirms its relevance, and determines the correct solution for any compact initial condition. The techniques and results we discuss are applicable to a wide range of similarity problems.

On vector-valued versions of Grothendieck's theorem for p-summing operators

Georg Schlüchtermann (University of Munich)

We present some vector-valued versions of the classical results, essentially due to Grothendieck, which say that every operator from $L^1(\mu)$ into ℓ^2 is absolutely summing and that every operator from $L^\infty[0,1]$ into $L^1(\mu)$ is 2-summing. We also show that the Banach spaces which share this last property with $L^1(\mu)$ are exactly those having cotype 2. In addition, all Banach spaces which satisfy the Grothendieck theorem are of cotype 2.

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Toral automorphisms and antiautormorphisms of rotation C^*-algebras

Peter Stacey (La Trobe University)

A rotation C^*-algebra is the universal C^*-algebra generated by two unitaries U, V satisfying $VU = \rho UV$ for some complex number ρ of modulus 1. If a, b, c, d are integers and λ, μ are complex numbers of modulus 1, then an automorphism ϕ of the algebra is determined by $\phi(U) = \lambda U^a V^c$ and $\phi(V) = \mu U^b V^d$ when $ad - bc = 1$. When $ad - bc = -1$ these formulae determine an antiautomorphism. The talk will discuss work of Hu Yaohua and the speaker on the classification of such automorphisms and antiautomorphisms up to conjugacy by arbitrary automorphisms.

Equality structures

Timothy Stokes (Murdoch University)

Generalising the *if-and-only-if* connective of Boolean algebra, we consider universal algebras which are in particular monoids and which have a binary operation $=_i$ of *internalised equality*, satisfying natural reflexivity and replacement rules.

More precisely, $\langle A, \cdot, =_i, \ldots \rangle$ is an *E-structure* if $\langle A, \cdot \rangle$ is a monoid with identity 1, with $=_i$ a binary operation satisfying

(1) $(x =_i x) = 1$,

(2) $x \cdot (y =_i z) = (y =_i z) \cdot x$, and

(3) $f(x) \cdot (x =_i y) = f(y) \cdot (x =_i y)$ for all derived unary operations f on A.

E-structures have importance in automated reasoning: for instance, we recover the usual models of standard S_4 modal logic by letting A be a Boolean algebra. However, E-structures abound across mathematics and have independent algebraic interest. Important varieties of E-structures include E-semilattices (representable in terms of topological spaces) and E-rings (equivalent to rings with a generalised interior operation). We characterise congruences on E-structures in terms of "normal" substructures and obtain a subdirect product representation of E-structures in which the semilattice of assertions is a Boolean algebra.

Combinatorics of representations of $\widehat{\mathrm{sl}}_n$, and a connection with Ariki-Koike algebras

T. A. Welsh (University of Melbourne)

We describe the crystal graph of highest weight representations of $\widehat{\mathrm{sl}}_n$ in terms of multipartitions, thereby obtaining a combinatorial expression for the characters of these representations. We characterise the subset of these multipartitions which decompose the tensor product of two such representations, and make a connection between these and representations of Ariki-Koike algebras (these algebras include the A and B type Hecke algebras as special cases) that are labelled by the same multipartitions.

Inversion of a generalised Hilbert transform

E. O. Tuck (Adelaide)

An integral transform $\mathcal{H}_y$ is defined which reduces to the ordinary Hilbert transform $\mathcal{H}_0$ when $y = 0$, and is useful in some hydrodynamic applications. Although $\mathcal{H}_y$ does not seem to be explicitly invertible for $y \neq 0$ (in contrast to $\mathcal{H}_0^{-1} = -\mathcal{H}_0$), it is readily invertible numerically for y less than a certain bound.

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