Partial Differential Equations Special Session

Australian Mathematical Society Annual Meeting
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Titles and abstracts of the talks

Speakers

- Athanassenas
- Grotowski
- Grüter
- Hassell
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Behaviour of singularities of the volume-preserving mean curvature flow

Maria Athanassenas (Monash University)

We use a geometric evolution process to construct surfaces of constant mean curvature, i.e. minimizing surface area and enclosing a given volume, with free boundaries. The main difficulty arises from the average mean curvature appearing in the equations describing the flow, due to the volume constraint. Standard (local) parabolic techniques do not apply and have to be replaced by a combination of purely geometric with analytical methods. In the case of rotationally symmetric surfaces we are able to control the average mean curvature, even when the flow develops singularities, so that we can describe their behaviour depending on the initial data.

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Partial regularity for nonlinear elliptic systems

Joseph Grotowski (Humboldt University, Berlin)

We consider quasilinear elliptic systems of divergence type, with inhomogeneity obeying the natural growth condition. We provide a new proof of partial regularity for weak solutions, based on a generalization of the technique of harmonic approximation.

This is joint work with Frank Duzaar

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Regularity results for parametric minimal surfaces and harmonic maps with a free boundary

Michael Grüter (University of Saarbrücken, Germany)

Parametric minimal surfaces in $\mathbb{R}^3$ are conformally parametrised mappings from a two-dimensional domain in to $\mathbb{R}^3$ such that each coordinate function is harmonic. Free boundary and partially free boundary problems for minimal surfaces have been studied since Courant's famous monograph appeared in 1950. In particular, the regularity problem has attracted the attention of a number of mathematicians ever since. More generally, the corresponding regularity question for harmonic mappings has been considered in recent times. In this talk, I want to give a survey of the main results attained so far and report on the latest progress.

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The spectral projections and resolvent for the Laplacian on a class of complete Riemannian manifolds

Andrew Hassell (Australian National University)

This is joint work with A. Vasy. We consider the Laplacian on a manifold with boundary X with 'scattering metric'. This is a form of metric which makes the interior of X a complete, asymptotically flat manifold which asymptotically looks like the large end of a cone; flat Euclidean space is an example. We construct the integral kernels of the spectral projections and the resolvent for the Laplacian as 'Legendrian distributions' (functions with oscillatory behaviour at the boundary). Using this, one can analyze other functions of the Laplacian such as the Schrödinger kernel.

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Multiple solutions of semilinear elliptic equations

H. B. Thompson (Queensland)

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We consider the semilinear elliptic boundary value problem

$$\Delta u + f(x, u, \nabla u) = 0, \quad x \in \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain. We show that there are at least three solutions provided there are suitable pairs of sub and super solutions and $f(x, u, p)$ does not grow too rapidly with respect to p. In the special case $f(x, u, \nabla u) = f(u) \geq 0$ we give upper and lower bounds on f, for various ranges of values of u which are related to the geometry of Ω, and apply our general result to show there are at least three positive solutions. We give an example showing this latter result is sharp.

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On the Bernstein problem for maximal affine hypersurfaces

Neil Trudinger (Australian National University)

The Bernstein problem for maximal affine hypersurfaces, as posed by Chern and Calabi, asks whether a locally uniformly convex function in Euclidean n-space, whose graph locally maximizes affine area in $(n+1)$-space must be a quadratic polynomial. The Euler equation is a nonlinear elliptic equation of fourth order. In this talk, I report on research with X.J Wang, which provides the affirmative answer in the case of two dimensions, thereby confirming the original conjecture of Chern. For higher dimensions, the result is established under a "uniform strict convexity" condition.

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Self-similar solutions of Gauss curvature flows

John Urbas (Australian National University)

I will describe some recent work on the classification of complete non-compact homothetic solutions of Gauss curvature flows.

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On the design of a reflector antenna

Xu-Jia Wang (Australian National University)

We consider a problem arising in astrophysics. For a given (not necessarily round) area in outer space, we want to construct an antenna to receive signals from the area. Mathematically this problem is equivalent to solving a Monge-Ampere type equation subject to a natural boundary condition. We prove the existence, uniqueness, and regularity of solutions to the problem.

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Singularly perturbed elliptic problems in exterior domains

Shusen Yan (University of Sydney)

In this talk, I shall present some of the recent results on the singularly perturbed problems in exterior domains with Dirichlet boundary condition or Neumann boundary condition. The work is mainly concerned with the effect of the domain topology on the existence of multipeak solutions. This is joint work with E. N. Dancer.

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On the quasiconvex hull

Kewei Zhang (Macquarie University)

We give a survey of results obtained recently on quasiconvex hulls. In some models of the variational approach to material microstructures, the problem is to understand the oscillation of a sequence of gradients Du_j for vector valued mappings u_j approaching a compact set K in a space of matrices, $\text{dist}(Du_j, K) \to 0$ a.e.. The quasiconvex hull $Q(K)$ of K records the weak limit of the sequence $Q(K)$ in the sense that if Du is the weak limit of Du_j in a Sobolev space, then $Du \in Q(K)$ a.e.. In this talk we will list some of the known properties of $Q(K)$, including a Krain-Milman type theorem, a stability theorem for the two well problem in martensitic phase transitions, and some known explicit examples. We also introduce other related semiconvex hulls and examine the relations among them.

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