MATH 402 Practice questions
Monday, 3 December, 2018

Exercise 1. Let ℓ be a hyperbolic line, with P a point not on ℓ.
  (1) Define the following terms:
    (a) A line m which is *limiting parallel* to ℓ through P; and in that case, the *angle of parallelism* of ℓ at P.
    (b) A line m through P which is ultraparallel to ℓ.
  (2) Prove that if ℓ and m have a common perpendicular, they must be ultraparallel.
  (3) Prove that ℓ and m cannot have more than one common perpendicular.

Exercise 2. Let $ABCD$ be a Lambert quadrilateral with right angles at A, B, C.
  (1) Draw a picture of the Lambert quadrilateral. Label the right angles. What do you know about the angle at D? What do you know about opposite sides of the Lambert quadrilateral (e.g. $\overline{AB}$ and $\overline{CD}$, or $\overline{BC}$ and $\overline{AD}$)?
  (2) Extend the side $\overline{BC}$ in both directions to points E (on $\overrightarrow{CB}$) and F (on $\overrightarrow{BC}$) such that $BE = CF$. Draw this on your picture.
  (3) Prove that the angles $\angle BEA$ and $\angle CFD$ are not congruent.

Exercise 3. In hyperbolic geometry, there is a regular tiling of type $(4, 6)$, in which each tile is a regular quadrilateral.
  (1) How many tiles meet at each vertex?
  (2) If we divide a quadrilateral tile into 4 congruent isosceles triangles meeting at the centre of the quadrilateral, what will be the base and summit angles of each isosceles triangles? (Justify your answer.)

Exercise 4. Suppose that $P = (0, a)$ is a point in the Klein model, with $a > 0$. Let (x, y) be the corresponding coordinates in Poincaré geometry.
  (1) How do you know that $x = 0$?
  (2) Which is true about y?
    • $y < 0$
    • $y = 0$
    • $y \in (0, a)$
    • $y = a$
    • $y \in (a, 1)$.

Exercise 5. Let ΔABC be an equilateral triangle with interior angle measure $45°$ (in hyperbolic geometry). Suppose that its area is 180 units. Let ΔXYZ be an isosceles triangle with summit angle $90°$ and base angles each $40°$. Calculate the area of this triangle.

Exercise 6. Are the following statements true or false?

(a)	Limiting parallels exist in the Klein model but not the Poincaré model.	True	False
(b)	Pasch's axiom is true for omega-triangles, but it's not an axiom anymore.	True	False
(c)	A limiting parallel is a special kind of ultraparallel.	True	False
(d)	Every line has exactly two omega points.	True	False
(e)	A Saccheri quadrilateral has exactly two congruent sides.	True	False
(f)	AAA congruence is a theorem in hyperbolic geometry, but not in Euclidean geometry.	True	False
(g)	A finite symmetry group which has $2n$ elements is dihedral.	True	False
(h)	It is possible to cut a Lambert quadrilateral into two Saccheri quadrilaterals.	True	False
(i)	Given two omega points, Ω_1, Ω_2 there is exactly one line which contains both of them.	True	False