MATH 402 Homework 2

Due Friday, September 14, 2018

Exercise 1. Consider the following statement (often known as the *Supplementary Angle Theorem*).

Theorem 1. *If a straight line is set up on another straight line to make two angles, the sum of these angles is equal to two right angles.*

a. *[3 points]* Draw a picture illustrating this result.
b. *[5 points]* Use this theorem to prove the Vertical Angle Theorem: *if two straight lines intersect, two non-adjacent angles formed by the intersection ("vertical angles") are congruent.*
c. For the next part of the question, you will use the Vertical Angle Theorem and Euclid's Proposition 27 (Theorem 2.7 in the book) to prove Euclid's Proposition 28, which has two parts.
   i *[5 points]* Assume that a line n falls across two lines ℓ and m such that the corresponding angles are congruent. Prove that ℓ and m are parallel.
   ii *[5 points]* Assume that a line n falls across two lines ℓ and m such that the sum of the measures of two interior angles on the same side is equal to two right angles. Prove that ℓ and m are parallel.
d. *[5 pts]* Let ΔABC be a triangle. Use the Exterior Angle Theorem and the Supplementary Angle Theorem to show that that the sum of any two interior angles is less than $180°$.

(Notice that none of these results use the Parallel Postulate—they hold in neutral geometry.)

Exercise 2. Consider a triangle ΔABC.

a. *[6 points]* Show how you can construct a parallel to $\overleftrightarrow{BC}$ through the point A. (You may use results on constructing perpendiculars without proof.)
b. *[6 points]* Use this to show that Playfair's Postulate (or Euclid's Parallel Postulate; or some other result that we've proven using one of these) implies that the sum of the angles in the triangle is equal to two right angles (i.e. $180°$).

Exercise 3. a. *[5 points]* Prove that SAS congruence implies ASA congruence. That is, assume that you know that SAS congruence holds. Assume that you are given a pair of triangles ΔABC and ΔXYZ such that they have the angles at A and X congruent, and the angles at B and Y congruent, and also the sides $\overline{AB}$ and $\overline{XY}$ congruent. Prove that the triangles are congruent.
b. *[5 points]* State and prove a SASAS congruence relation for quadrilaterals. *Hint: draw lines to divide your quadrilaterals into triangles, and then prove use triangle congruence results.*

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.