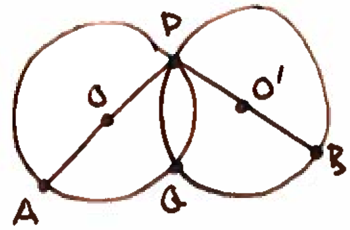
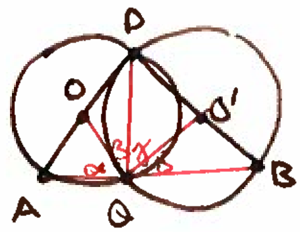
MATH 402 – HW4 Solutions.

①



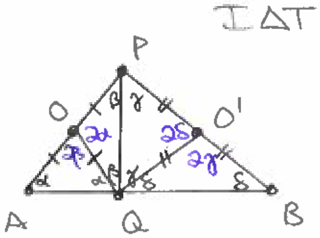
(a) draw rays from Q to A, O, P, O', B and label the angles by $\alpha, \beta, \gamma, \delta$.



(b) Determine the angles of the four triangles with Q as a vertex.

• note that many of the line segments are radii of the circles O or O'

$\Rightarrow$ we have 4 isosceles triangles and can use the IΔT to label the matching base angles



• Also, the angles at O, O' are central angles so their measure is twice that of the corresponding inscribed angles (shown in purple above.)

(c) Prove that $\alpha+\beta+\gamma+\delta=180°$ and conclude that $Q \in \overleftrightarrow{AB}$

we have many ways to see that $\alpha+\beta=90°$

① since $\angle AQP$ is an inscribed angle along a semicircle, is is $90°$.

$$\therefore \alpha+\beta=90°$$

② looking at point O, the supplementary angle theorem tells us that $2\alpha+2\beta=180°$

$$\Rightarrow \alpha+\beta=90°$$

③ looking at $\triangle AOQ$ or $\triangle POQ$, we have angle sum

$$2\alpha+\beta+\beta = \alpha+\alpha+2\beta = 180°$$

$$\Rightarrow \alpha+\beta = 90°$$

Likewise $\gamma+\delta = 90°$

$\therefore$ $\alpha+\beta+\gamma+\delta = 90°$

i.e. the ray $\overrightarrow{QB}$ forms angle 180° with $\overrightarrow{QA}$

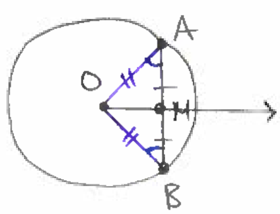
but the ray at Q that we get by extending the line $\overleftrightarrow{QA}$ also forms angle 180° (by supplementary angle theorem)

$\therefore$ this ray is equal to $\overrightarrow{QR}$

i.e. $\overleftrightarrow{AR} = \overleftrightarrow{AQ} = \overleftrightarrow{QR}$.

Exercise 2: c a Euclidean circle with centre O.
A, B two points on the boundary of c s.t. $\overline{AB}$ is a chord but not a diameter.

(a). let M be the midpoint of $\overline{AB}$. Prove that $\overrightarrow{OM}$ is perpendicular to $\overline{AB}$.



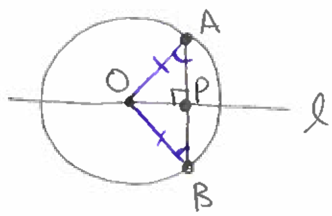
① $OA = OB$ because they are both radii.

② $\therefore$ $\triangle OAB$ is an isosceles triangle, so by the isosceles triangle theorem $\angle OAM \cong \angle OBM$.

③ SAS congruence $\Rightarrow$ $\triangle OAM \cong \triangle OBM$, and in particular $\angle AMO = \angle BMO$

i.e. $\overrightarrow{OM}$ is perpendicular to $\overline{AB}$.

(b) drop the perpendicular from O to $\overleftrightarrow{AB}$.
Prove it bisects $\overline{AB}$.



let P be the intersection point.

① As above, ΔOAB is isosceles, and the angles at A and B are congruent

② By AAS congruence, $\Delta OAP \cong \Delta OBP$.
and in particular $\overline{AP} \cong \overline{BP}$.