# MATH 402 Review for October 1–5

**Topics:** Hyperbolic geometry: Poincaré model and Klein model (7.1 and 7.2); Euclidean isometries (5.1). These were covered in lecture and in Worksheet 4. This material will also appear in Homework 5.

1. **Recall from last week:**

   (a) We used this theorem a lot this week:

   **Theorem 1** *Given a circle $c$ and two points $P$,$Q$ either in the circle or on its boundary, there exists a unique line (if $P$, $Q$ lie on a diameter of $c$) or circle (if not) which passes through $P$ and $Q$ and which is orthogonal to the circle $c$).*

   (b) We also needed to remember the construction of the inverse of a point with respect to a circle (which allows us to construct the *pole* of a chord).

2. **Things to know about hyperbolic geometry:**

   (a) Hyperbolic geometry is what we get when we replace Euclid's parallel postulate with the *hyperbolic parallel postulate*:

   **Hyperbolic Parallel Postulate 1** *Given a line $\ell$ and a point $P$ not on $\ell$ there exist at least two lines through $P$ parallel to $\ell$.*

   (b) The way that we know that this axiom does not contradict the first four of Euclid's axioms is that we can write down *models* in which Euclid's first four axioms and the hyperbolic parallel postulate all hold.

3. **Things to know about the Poincaré model:**

   (a) Definitions: Poincaré points, lines, angles, distance.

   (b) Make sure you know why each of the five axioms hold in this model.

4. **Things to know about the Klein model:**

   (a) Definitions: Poincaré points, lines, distance.

   (b) Make sure you don't get the two models mixed up.

   (c) Make sure you understand the definition of perpendicular lines in the Klein model.

   (d) Make sure you know why each of the five axioms hold in this model.

   (e) Definitions: limiting parallel, angle of parallelism.

5. **Things to know about transformations/isometries:**

   (a) Definitions: transformation, injective, surjective, bijective, (Euclidean) isometry.

   (b) Results: Isometries are injective and preserve line segments and lines. Isometries form a group.

**Practice Questions**

1. **Practice with the Klein model and the Poincaré model:**

   - Practice using the distance functions. Choose coordinates for two points in the Klein model and find the distance between them. (This is a bit harder to do in the Poincaré model because it is harder to find the coordinates of the points $R$ and $S$ that we need for the formulas—why is this?)
   - Practice drawing perpendicular lines in the Klein model. See if you can check the following statement: if $\ell$ is perpendicular to $m$, then $m$ is perpendicular to $\ell$. (There are different cases to check, depending if $\ell$ or $m$ are diameters or other chords.)