MATH 402 Homework 6

Due Friday 10/14/16

(1) (*10 pts.*) Recall from the last worksheet that a *semiregular* tessellation is one made from copies of two or more regular polygons, such that the configuration around each vertex is the same (up to rotation). Show that there can be no semiregular tessellations with regular pentagons, squares, and equilateral triangles.

(2) (*10 pts.*) Prove that two hyperbolic lines in the Poincaré disk can have at most one point of intersection.

(3) (*15 pts.*) We would like to know that Hilbert's betweenness axioms are satisfied by the Poincaré model of hyperbolic geometry. Define betweenness in the Poincaré model. Then, translate the betweenness axioms into statements that should hold in Euclidean geometry.

(4) (*10 pts.*) Consider the Poincaré hyperbolic disk defined by a circle $\mathcal{C}$ with center O. Recall that distance is defined as follows. If we want to compute $d_p(P,Q)$, let $\mathcal{S}$ be the hyperbolic line containing P, Q. We need to find the points R, S in the intersection of $\mathcal{S}$ and $\mathcal{C}$. Then we define

$$d_p(P,Q) = \left| \ln \frac{(\overline{PS})(\overline{QR})}{(\overline{PR})(\overline{QS})} \right|.$$

(a) Show that $d_p(P,Q) = 0$ if and only if $P = Q$.

(b) What is the distance between a point P and the "origin" O?

(c) How does $d_p(P,Q)$ change as we keep P fixed and move Q (in a hyperbolic line) towards $\mathcal{C}$?

(d) Draw several hyperbolic circles, some of which have centers close to O, and others which have centers close to $\mathcal{C}$.

(5) (*5 pts.*) Euclid second postulate said that lines can always be extended. Is this true in the hyperbolic disk? Why?