MATH 595 Tuesday 17 April

More practice with embeddings of curves; the canonical embedding

(1) **Exercise IV.3.6** *Curves of degree 1*
(For this question, you can use the results of exercises 3.4 and 3.5 without proof.)
   (a) If X is a curve of degree 4 in some $\mathbb{P}^n$, not contained in any hyperplane, show that we must be in one of the following situations:
      (i) $g = 0$: in this case X is either the rational normal quartic in $\mathbb{P}^4$ or the rational quartic curve in $\mathbb{P}^3$;
      (ii) $X \subset \mathbb{P}^2$: in this case $g = 3$; or
      (iii) $X \subset \mathbb{P}^3$: in this case $g = 1$.
   (b) In the case $g = 1$, show that g is a complete intersection of two irreducible quadric surfaces in $\mathbb{P}^3$.
   (Hint: use the exact sequence associated to $X \subset \mathbb{P}^3$ to compute the dimension of $H^0(X, \mathscr{I}_X(2)$, and thus conclude that X is contained in at least two irreducible quadric surfaces.)

(2) **Exercise IV.5.1** Show that a hyperelliptic curve can never be a complete intersection in any projective space.