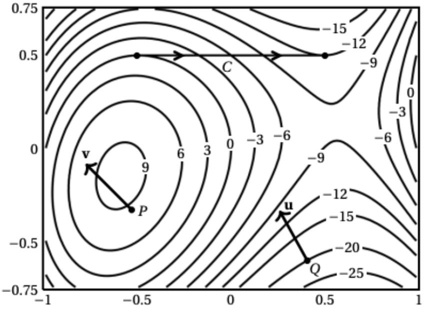
5. The contour map of a differentiable function g is shown at right. For each part, circle the best answer. **(2 points each)**



(a) The directional derivative $D_{\mathbf{v}}g(P)$ is:

positive	negative	zero

(b) The vector $\mathbf{u}$ is parallel to $\nabla g(Q)$.

True	False

(c) Estimate $\int_C g(x,y)\, ds$.

−12	−9	−6	−3	0	3	6	9	12

(d) Find $\int_C \nabla g \cdot d\mathbf{r}$:

−12	−9	−6	−3	0	3	6	9	12

The above problem is from Midterm 2 in Fall 2018. Use your i-clicker to vote on which part of the problem you'd like us to go over right now. (Vote (e) if you don't want to go over any of it.)

Midterm Announcements

- Midterm 2 is next Tuesday, 12 March, beginning at 7:15pm. Please arrive by 7pm.
  - Exam location is posted on the Midterm 2 webpage, and is based on your discussion section.
- Some tips on what to expect on the midterm:
  - The emphasis is on Calc III material, not Calc I & II material. Use your time wisely; if you get stuck on a complicated integral, go back and see if you made a mistake in setting up the equations, or move on to another problem.
  - There will be multiple choice questions where more than one answer is correct, or where none of the answers are correct. Read the instructions carefully; they say explicitly how many choices you must/are allowed to make.
  - There will be questions where there is more than one way to solve the problem. Read the instructions carefully; they may tell you which method you must use (in which case points will not be given for other methods).

Review of integration over an interval

Consider a function g on $[a, b]$.

Divide $[a, b]$ into n equal pieces $[x_{i-1}, x_i]$ of width $\Delta x = \frac{b-a}{n}$. Pick any $x_i^* \in [x_{i-1}, x_i]$, for each i.

Define

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} g(x_i^*)\Delta x$$

if the limit exists and doesn't depend on the choices of x_i^*.

Review of integration over an interval

Theorem

If g is bounded on $[a, b]$ and continuous except at a finite number of points, then $\int_a^b g(x)ds$ is well-defined.

Practice with the midpoint rule

Let $D = [0,4] \times [1,5]$ and let $f(x,y) = x + y$.

Use the midpoint rule with $m = n = 2$ to estimate $\iint_D f dA$.

(a) 0

(b) 10

(c) 20

(d) 80

(e) I don't know

Solution

We identify our four points (x_{ij}^*, y_{ij}^*) as follows:

$$(1,2),(3,2),(1,4),(3,4).$$

Also notice that $\Delta A = 2 \times 2 = 4$ in this case.

So the midpoint rule becomes

$$\begin{aligned}\iint_D (x+y)dA &\approx [f(1,2)+f(3,2)+f(1,4)+f(3,4)]\,\Delta A \\ &= [3+5+5+7]\,(4) \\ &= 80.\end{aligned}$$

Practice with iterated integrals

Let $D = [0, 2] \times [-3, 1]$. Find $\iint (3x^2 + 3y^2) dA$.

(a) -12

(b) 42

(c) 88

(d) Some other number

(e) I don't know

(If you're done, try integrating using the opposite order of integration to what you used the first time. You should get the same answer.)

Solution

$$
\begin{aligned}
\iint_D (3x^2+3y^2)\,dA &= \int_0^2 \int_{-3}^1 (3x^2+3y^2)\,dy\,dx \\
&= \int_0^2 \left[3x^2y + y^3\right]_{-3}^1 dx \\
&= \int_0^2 \left[(3x^2+1) - (-9x^2-27)\right] dx \\
&= \int_0^2 12x^2 + 28 \; dx \\
&= [4x^3 + 28x]_0^2 \\
&= 32 + 56 = 88.
\end{aligned}
$$

Solution (opposite order)

$$\begin{aligned}\iint_D (3x^2+3y^2)dA &= \int_{-3}^{1}\int_0^2 (3x^2+3y^2)dxdy\\ &= \int_{-3}^{1}\left[x^3+3xy^2\right]_0^2 dy\\ &= \int_{-3}^{1}\left[8+6y^2\right] dy\\ &= \left[8y+2y^3\right]_{-3}^{1}\\ &= (8+2)-(-24-54)\\ &= 88.\end{aligned}$$