Thursday, January 17 $**$ *Parametric curves defined using vector arithmetic.*

1. Let $f(x) = x^2 + x - 2$.

   (a) Graph the equation $y = f(x)$. (By hand, then check with a calculator if you want.)

   (b) Find the slope and equation of the tangent line to $y = f(x)$ when $x = 2$. Draw the tangent line on your picture.

   (c) Draw a vector in $\mathbb{R}^2$ that describes the direction of the line. Find a numeric representation of your vector.

2. Consider the curve given parametrically by

$$\begin{cases} x(t) = t \\ y(t) = t^2 + t - 2 \end{cases} \qquad \text{for } 0 \le t < 4.$$

   (a) Sketch the curve. How does this graph differ from your graph in Problem 1(a)?

   (b) Consider the vectors formed by the pair $\big(x(t), y(t)\big)$. Anchoring the vectors at the origin, sketch on your graph the vectors at time $t = 0, 1, 2, 3$.

   (c) Now consider the vectors formed by $\big(x'(t), y'(t)\big)$. Evaluate $\big(x'(t), y'(t)\big)$ at time $t = 2$, what does the vector represent? Hint: Graph it on the curve at the point $\big(x(2), y(2)\big)$.

   (d) Imagine that the curve is the path of a moving particle. What is the speed of the particle when $t = 2$?

3. (a) Sketch the vector emanating from the origin ending at the point $(-5, 2)$ in $\mathbb{R}^2$.

   (b) On the same graph and using the "head-to-tail" geometric addition method, draw the vector $(-5, 2) + (3, -1)$.

   (c) Do the same for $(-5, 2) + 2(3, -1)$.

   (d) Do the same for $(-5, 2) + (-1)(3, -1)$.

   (e) If we allow the scalar mulitplying the vector $(3, -1)$ to vary, what geometric object is described by the parametric equation $(-5, 2) + t(3, -1)$ for all t?

4. Consider the set of points in $\mathbb{R}^3$ defined by the parametric equation

$$\mathbf{l}(t) = (-5 + 2t, 2 + 3t, 1 - t) \quad \text{for all } t.$$

   (a) Using the properties of vector arithmetic, factor $\mathbf{l}(t)$ into the form $\mathbf{p} + t\mathbf{v}$ where $\mathbf{p}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$.

   (b) Using the factored form (and your technique from Problem 3) sketch this object in $\mathbb{R}^3$. Geometrically, what does this parametric equation describe?

   (c) Why is the vector $\mathbf{v}$ in your factored form referred to as the *direction vector*?

5. Let $\mathbf{a} = (-\sqrt{3}, 0, -1, 0)$ and $\mathbf{b} = (1, 1, 0, 1)$ be vectors in $\mathbb{R}^4$.

   (a) Find the distance between the points $(-\sqrt{3}, 0, -1, 0)$ and $(1, 1, 0, 1)$.

   (b) Find the angle between $\mathbf{a}$ and $\mathbf{b}$.