Errata and Addenda for "Algebraic Invariants of Links"

9 December 2010

New references and substantial changes are starred.

My thanks to Sergey Melikhov for his observations [June 2008].

Chapter 1:

*page 5 line -8: "weight" was not defined! The weight of a group G is the minimal number of elements in a subset S such that $\langle\langle S\rangle\rangle_G = G$.

page 5 line -1: "group group" should be "group".

page 6 line -10: "conection" should be "connection".

page 9 line 1: "$\mathscr{2}$" should be "2".

page 10 line 4: See Theorem 2.2 of [Bir] for this assertion.

page 13 line -10: "successively" is misspelt.

*page 17, lines 14-16: replace these two sentences by "The links L and $h(L)$ are then said to be obtained from each other by an *elementary surgery*".

*page 18, line 5: "$T_i(sz, s)$" should be "$T_i(s^{\pm 1}z, s)$". (The oversight here affected also Theorem 6.11 and 7.1).

*page 18, lines 22-25: No. the ribbon group associated to such a ribbon has a 2-generator, 1-relator presentation, and so the knot module of a ribbon knot bounding a ribbon of this form can be generated by 2 elements. Counterexample: $(3_1\# - 3_1)\#(3_1\# - 3_1)$. I suspect that 9_{41} may also be a counterexample, as it bounds a ribbon disc with $Z/3Z$ symmetry.

*page 21 line 8: replace the sentence beginning on this line with "Thus Example 12 of [Fo62] is not ribbon *[Yj64] (see also Theorems 4.3 and 4.6 below), although it is slice [Ke65]."

page 23 line -8: add ". See Figure 3(a) of [Wa94]" before the ")".

Chapter 2:

page 32 line 11: "$I_q(u, \partial v)$" should be "$I_{q+1}(u, \partial v)$".

page 32 line -7: the subscript "$g \in Z^\mu$" should be "$g \in \pi/H$".

*page 34 (proof of 2.1): the arguments here deserve more explanation. The Snake lemma shows that $t-1$ is injective on T and that $T/(t-1)T \leq Z^{\mu-1}$. Since T is a noetherian torsion Λ-module it follows that $T = (t-1)T$. The observation that H_{n+1} is free should have been made before estimating its rank. Since an onto endomorphism of a noetherian module (such as H_n) is an isomorphism, $Z \otimes H_{n+1} \cong Z^{\mu-1}$.

page 36, line 15: "$\theta_p D_p$" should be "$\theta_{2-p} D_{3-p}$".

page 36, line 20: Λ_1 should be Λ_μ throughout this exact sequence.

*page 36, lines -11 to -9: "link module sequence" is defined in Chapter 4.

*pages 38, line 11: "Then P" should be "Then $P^\perp$", and the subsequent argument should be expanded slightly, The existing argument shows that $P^\perp/P$ is pseudonull. Since $S(\Lambda_\mu)$ has no nontrivial pseudonull submodule it then follows that $P^\perp = P^{\perp\perp}$. (My thanks to Jae Choon Cha and Tim Cochran for alerting me to a gap in the proof of this Theorem.)

page 38, line 18: "X'" should be "Z'".

page 38, line -11: "d" should be "∂".

page 39 lines 12, 13: "C_n to ... $(-1)^{n+1}$ " should be "C_{2q+1} to ... $(-1)^q$ ".

page 42 line -12: "b_N" should be "$-b_N$".

Chapter 3:

*page 47 line -5: "$r \in M$" should be "r some nonzero divisor of R".

*page 53 line 22: "Chapter 6" should be "Theorem 3.22 below".

*page 57 lines 18-20: The definition of resultant in this sentence is not correct. However the next sentence is correct and the resultant is used correctly below. (See [Lan], page 135). If θ is monic and $M = R[u, u^{-1}]/(\theta, \psi)$, considered as an R-module then it may easily be shown that $E_0(M) = (Res_R(\theta, \psi))$.

*page 62 lines 2-9: "r" should be "$q - r$" throughout this proof, except in the final sentence.

Chapter 4:

*page 71 line 9: there is no need to invoke Hilbert's Syzygy Theorem here, for if M is a $R\Lambda_\mu$-module which is free as an R-module and $K(\Lambda_\mu)_*$ is the Koszul complex given below then $M \otimes_{\mathbb{Z}} K(\Lambda_\mu)_*$ with the diagonal Z^μ-action is a free resolution of M of length μ. In general, if $\phi : F \to M$ is an epimorphism from a free $R\Lambda_\mu$-module F then splicing such a resolution of $\mathrm{Ker}(\phi)$ with ϕ gives a free resolution of M of length $\mu + 1$.

page 74, lines 15-20: when $\mu = 1$ (the knot-theoretic case) I_μ is free. In this case we may obtain a square presentation matrix for B by adding a relation to kill the generator of A/B, without using "projectives are free".

*page 80 line 12: we should assume here that $\alpha(L) = \mu$.

page 82 line 13: "*0*" should be "0".

page 86 line -10: "sparated" should be "separated".

page 91 line 7: "*0*" should be "0".

*page 92 lines 5-8: this deserves more explanation. The corresponding result for field coefficients follows easily from determinantal characterizations of the rank. In the integral case one reduces firstly to the $\mathbb{Z}$-torsion submodules and then to the p-primary torsion, and then inducts on n, where p^n is the exponent of this subgroup.

Chapter 5:

page 95, line -3: "follwing" should be "following".

*page 96 lines 15 to 17: this is not clear if $\mu > 1$, as in general $\Theta A \neq A\Theta$.

page 99 line 17: "(*ii*)" should be "the second Torres condition".

§5.6: see *[Hi05] for an account of the singularities of plane curves using little more than [AM].

*page 112 line 4: $\mu(f)$ is called the "Milnor number", not the "multiplicity".

*page 112 line 16: insert "for m sufficiently large" before "by Theorem 13.6 of [EN]".

*page 114 lines -6, -5: Here $R = (t-1)H$, as in Theorem 5.10.

*page 114 line -5: "H" should be "$H/(t^k-1)H$".

*page 119 lines 3-8: see *[Po04] for an improvement upon [MM82], using Reidemeister-Franz torsion.

page 120, line -5: $t - t^{-1}x$ should be $(t-t^{-1})x$.

page 124 line 10: we may delete "If $\rho \neq \varepsilon_F$", for otherwise f and f_0 factor though π/π', and so are equal.

*page 124 line 12-13: The function is not onto if $\mu = 2$. Thus "if $\mu \leq 2$. ... $\mu \geq 3$." should read "in the knot-theoretic case [deR67]. However this map is no longer onto if $\mu \geq 2$."

§5.10: this section can be substantially improved. The key point is that $V \otimes_K - = W \otimes_\pi -$, where $W = V \otimes_K R[\pi]$ is an $(R[\pi/K], R[\pi])$-bimodule, and is finitely generated and free as a left $R[\pi/K]$-module. Hence twisted invariants may be derived from cohomology with local coefficients. From this point of view it is easy to show that twisted Alexander polynomials of symmetric knots satisfy a Murasugi formula *[HLN06].

*page 125 line -11: "The elementary" should be "When $C_* = C_*(\tilde{X})$ the elementary".

*page 125 line -8: the notation $H_*(X;\alpha,V)$ was not defined. It is of course the homology of the complex $V \otimes_{R[K]} C_*(\tilde{X})$.

page 127 line 15: "coovering" should be "covering".

Chapter 6:

*page 134 line-10: "Theorem 7 of [Mat]" should be "Theorem 29 of [Mat]".

*page 143 line -7: insert "$\varepsilon(\mathbb{D})$" between "$\mathbb{D}^{-1}$" and "u".

*page 143 line -1: "$T_i(s,s)$" should be "$T_i(s^{e(i)},s)$, for some $e(i) = \pm 1$ and".

*page 145 lines 4 and 5: these should read

$= \Sigma\Sigma[\mathbb{D}^{-1}]_{kj}\delta_{ik}\delta_{n0}e(i)t^n$

$= -[\mathbb{D}^{-1}]_{ij}e(i)$

page 147 line 6: 'Theorem 1" should be "Theorem 6.1".

page 147 line -9: "Lemma 11" should be "Lemma 6.12".

page 147 line -7: "M_k" should be "M_m".

Chapter 7:

*page 150, line 18: "$T_i(s,s)$" should be "$T_i(s^{e(i)},s)$, for some $e(i) = \pm 1$ and".

§7.6 should be clarified. In particular the auxiliary space W is used only to specify a framing. The Kervaire obstruction is always 0 *[Da05].

page 165 line -13: "$N(H)$" should be "$N(Ho)$".

page 165 line -10: "$X(H)$" should be "$X(Ho)$".

page 166 line 4: the notation is ambiguous; here $\Lambda_2 = \mathbb{Z}[\pi_1(T)]$.

*page 166 line -2: "$P \cup W$" should be "$P \cup_{X(Ho)} D^4$".

page 167 line 2: delete "(in $E \subset W$)".

page 167 line 4: "one" should be "on".

Chapter 8:

*page 172 line -3: insert "not" before "invertible".

page 174 line 20: if K is strongly $-$amphicheiral $\Delta_1(K)(t^2) = f(t)f(-t)$ by the argument of Theorem 8.16, and $f(t) \doteq \overline{f(t)} = f(-t^{-1})$ by duality.

page 174 line -13: delete "it".

page 179 line -7: insert "the" before "covering".

page 181 lines 5, 14: "$H_1(X; c^*\Lambda_\mu)$" should be "$H_1(X^\gamma;\mathbb{Z})$".

page 182 lines 8,9,-6,-3 and page 183 line 2: "λ" should be "ℓ".

page 182 line 13: Λ should be Λ_2

page 183 lines 11 and 14: "K" should be "$K_{m,n}$".

page 184 line -9: "$\widehat{\delta}$" should be "$\widehat{\Delta}$".

*page 187 lines 1 and 2: should read "The Alexander polynomial of $\#^3 4_1$ satisfies the Murasugi conditions with $\ell = 1$. This knot".

*page 187 line 3: "8.8" should be "8.10".

page 187, line -2: "by" should be "By".

*page 188, line 12: "$N =$" should be "$U \backslash$".

page 188, line -4: "then" should be "Then".

page 190 lines -6, -4: The initial letters of these sentences should be capitalized.

page 197 line -2: "$A \cap \mathcal{K}$" should be "$A \cap \mathcal{L}$".

page 198 line 18: "." should be ",".

*page 199 line -5: The argument of Theorem 8.22 may be refined to show that the denominator divides a power of q *[Hi04]. In fact we may assume that the denominator is 1 *[Ch04]. Cha observes that $\bar{A} \cup \bar{K}$ is concordant to Ho and so the Blanchfield pairing is neutral (without any localization).

Chapter 9:

*page 205 line 5: the result quoted from [FV92] was first proven by Dicks and Sontag, in *[DS78].

page 208, Lemma 9.4: "N" should be "M" throughout this lemma.

page 208 line -2: "))" should be ")".

page 209 line -3: "$\to\to$" should be "$\to$".

page 211 line 6: close the gap between "exterior" and ".".

*page 215 Theorem 9.11(6): delete "and only if". (The Tor group could be finite. I know of no such examples).

page 217 line -9: insert a space between "a" and "$(-1)^q$-linking".

*page 218 line 6: "part (4)" should be "parts (3) and (6)".

Chapter 10:

page 230 lines -5 and -4: this should be incorporated into "(5)" immediately above.

page 234 line -12:: "minimal model" should be "*minimal model*".

page 236 line 4: interchange ∂X_1 and ∂X_2 in the equation.

page 237 line -8: “in in” should be “is in”.

page 239 line 18: insert “part (4) of” before “Theorem 10.4”.

page 239 line 22: add “with distinct indices” at the end of this sentence.

*page 239 line -9: “$\binom{r}{s}$” should be “$\binom{r-1}{s}$”.

*page 240 line -12: the result (3) quoted here should read “$m(I)(\eta_{i_r}) = -m(I)(\eta_{i_1}) = (-1)^r\bar{\mu}(I)$ *for* $i_1 \neq i_r$, *and* $m(I)(\eta_j) = 0$ *otherwise*”.

page 241, line 7: “$\langle x_i\rangle$” should be “$\langle x_i\rangle$ in $Mil(G)$”.

page 241, line 8: “$G = \Pi A_i$” should be “$Mil(G) = \Pi A_i$”.

page 243 line -13: “1-link Ls” should be “1-links L”.

page 243 lines -8 to -5: the cited result of Cochran involves a different equivalence relation “$(2, k)$-cobordism”.

Chapter 11:

page 247 line -2: “$n > 1$” should be “$n \geq 1$”.

Biblio:

the thesis [She] was published as Memoir 784 of the AMS Memoir (2003).
[COT99] has appeared: Ann. Math. 157 (2003), 433-519.
[Mc01] has appeared: Ann. E.N.S. 35 (2002), 153-171.
[SW02] has appeared: Topology 41 (2002), 979-991.

Add:

*[Ch04] Cha, J.C. A characterization of the Murasugi polynomial of an equivariantly slice knot, arXiv:math.GT/0404403 v1 22 April 2004.

*[Da05] Davis, J.F. A two component link with Alexander polynomial one is concordant to the Hopf link, Math. Proc. Camb. Phil. Soc. 140 (2006), 265-268.

*[DS78] Dicks, W. and Sontag, E. D. Sylvester domains, J. Pure Appl. Algebra 13 (1978), 243-275.

*[Hi04] Hillman, J.A. Polynomials of equivariantly slice knots, available at http://www.maths.usyd.edu.au:8000/u/jonh/ (Feb 2004).

*[Hi05] Hillman, J.A. Singularities of plane algebraic curves, Expo. Math. 23 (2005), 233-254.

*[HLN06] Hillman, J.A., Livingston, C. and Naik, S. Twisted Alexander polynomials of periodic knots, Alg. Geom. Top. 6 (2006), 143-167.

*[Po04] Porti, J. Mayberry-Murasugi’s formula for links in homology 3-spheres, Proc. Amer. Math. Soc. 132 (2004), 3423-3431.

*[Yj64] Yajima, T. On simply knotted spheres, Osaka J. Math. 1 (1964), 133-152.