Assignment 5

This assignment should be returned by the beginning of the lecture on
Friday, 26 October 2007.

Q6 Determine a biholomorphic mapping from the upper half plane to the region

$$\{z \in \mathbb{C} | \Im(z) > 0, \Re(z) > 0, \min(\Im(z), \Re(z)) < 1\}.$$

Q7 Let $f(z) = \sqrt{z-1}\sqrt[3]{z-i}$.

(a) What are the branch points and what are their orders?

(b) Why is it not possible to define branches of f using a single branch cut connecting the branch points?

(c) How many values does $f(z)$ have at a generic point $z \in \mathbb{C}$?

Q8 (Gamelin I.7.7) Describe the Riemann surface associated with the function

$$f(z) = \sqrt{(z-x_1)\cdots(z-x_n)},$$

where $x_i \in \mathbb{R}$, $x_1 < \ldots < x_n$ and $n \geq 1$. (Hint: Consider n even or odd separately.)

Q9 Let $\omega \in \mathbb{C} \setminus \{0\}$, and let $\mathbb{Z}\omega$ be the set of all integral multiples of ω. Let S be the set of all congruence classes $z + \mathbb{Z}\omega$, $z \in \mathbb{C}$. Show that S is a Riemann surface which is conformally equivalent to the punctured plane $\mathbb{C} \setminus \{0\}$.

Q10 Let S be a Riemann surface and $f, g \colon S \to S$ be two holomorphic functions such that $f(s) = g(s)$ for all $s \in U$, where U is a non-empty, open subset of S. Show that $f(s) = g(s)$ for all $s \in S$.